A Complex-LASSO Approach for Localizing Forced Oscillations in Power Systems

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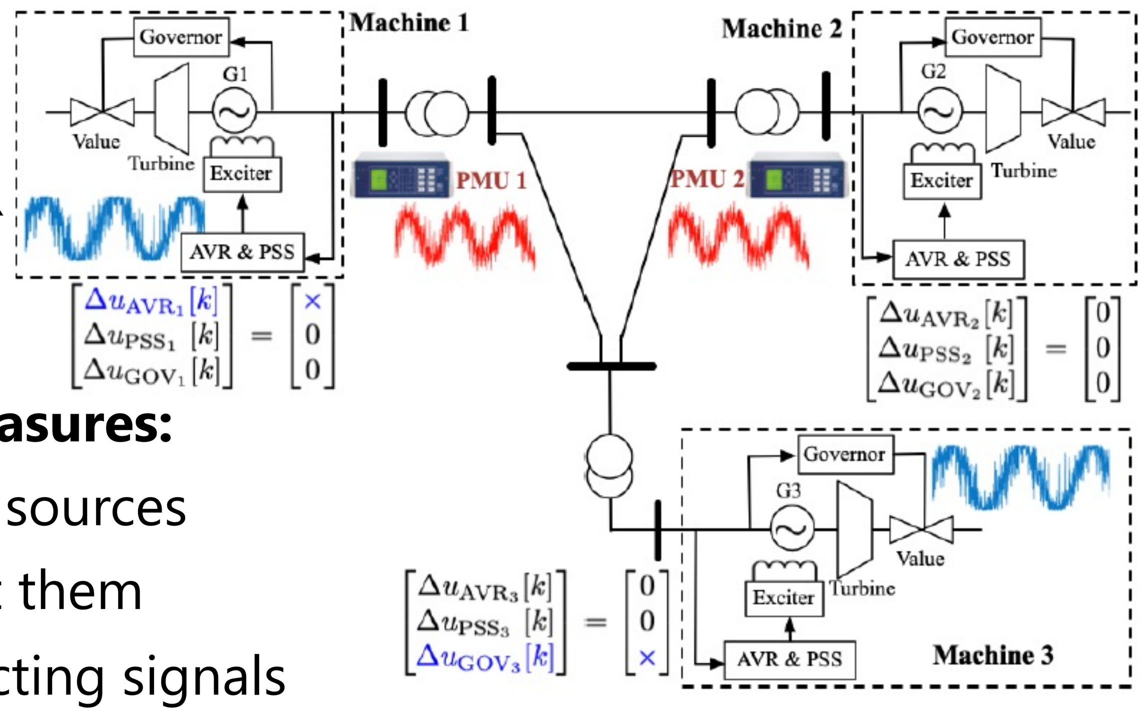
MOTIVATION

❖ Forced low-frequency oscillations (1-15 Hz) pose significant threat for security and stability of bulk-power systems

❖ **Sources:**
- malfunctioned controllers/AVR
- cyclic loads

❖ **Mitigation measures:**
- quickly localize sources and disconnect them
- inject counteracting signals



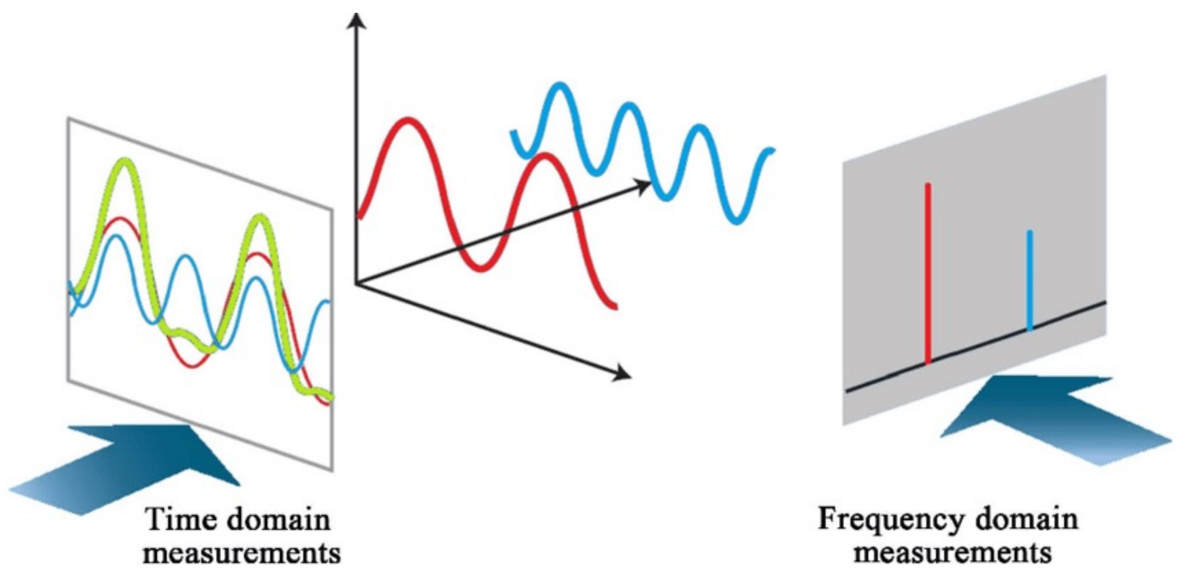
OBJECTIVE

❖ To design a data-driven algorithm that consider system dynamics and oscillation characteristics (e.g, sinusoids) to
- accurately localize oscillatory sources
- estimate input parameters (amplitude, frequency, phase)

KEY IDEA: spatial and temporal sparsity

-input in **time-domain** as sum of "unknown no. of. sinusoids"

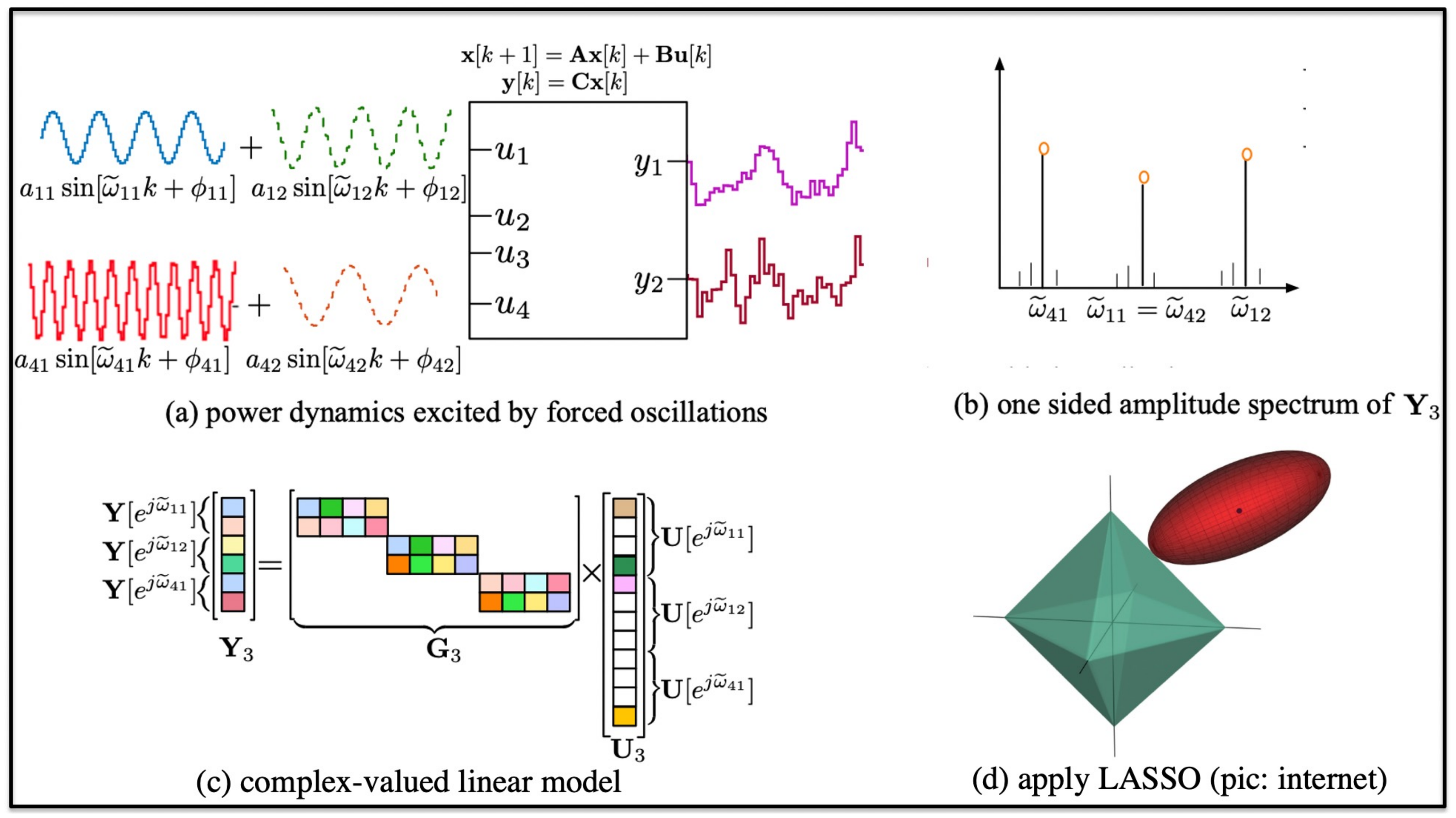
$$\mathbf{u}(t) \triangleq \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix} = \begin{bmatrix} \sum_{l=1}^{M_1} a_{1,l} \sin(\omega_{1,l} t + \phi_{1,l}) \\ \sum_{l=1}^{M_2} a_{2,l} \sin(\omega_{2,l} t + \phi_{2,l}) \\ \vdots \\ \sum_{l=1}^{M_m} a_{m,l} \sin(\omega_{m,l} t + \phi_{m,l}) \end{bmatrix}$$



-input in **frequency-domain** as sum of "unknown no. of. sinusoids"

$$\mathbf{U}[e^{j\Omega}] = j\pi \begin{bmatrix} \sum_{l=1}^{M_1} a_{1,l}\, e^{j\phi_{1,l}} [\delta(\Omega+\tilde{\omega}_{1,l}) - \delta(\Omega-\tilde{\omega}_{1,l})] \\ \sum_{l=1}^{M_2} a_{2,l}\, e^{j\phi_{2,l}} [\delta(\Omega+\tilde{\omega}_{2,l}) - \delta(\Omega-\tilde{\omega}_{2,l})] \\ \vdots \\ \sum_{l=1}^{M_m} a_{m,l}\, e^{j\phi_{m,l}} [\delta(\Omega+\tilde{\omega}_{m,l}) - \delta(\Omega-\tilde{\omega}_{m,l})] \end{bmatrix}$$

LOCALIZATION FRAMEWORK



System transfer function and DTFT

$$\mathbf{H}[z] = \mathbf{C}(z\mathbf{I} - \mathbf{A}_d)^{-1}\mathbf{B}_d \in \mathbb{C}^{p\times m}$$

$$\mathbf{Y}[z] = \mathbf{H}[z]\mathbf{U}[z] \quad \mathbf{Y}[e^{j\Omega}] = \mathbf{H}[e^{j\Omega}]\mathbf{U}[e^{j\Omega}]$$

Set of **K** input frequencies $\Omega \in \{\pm\tilde{\omega}_{1,1}, \ldots, \pm\tilde{\omega}_{m,M_m}\}$

$\tilde{\omega}_1$ components by all PMUs $Y[e^{j\tilde{\omega}_1}] \in R^{p\times 1}$

$$\underbrace{\begin{bmatrix} \mathbf{Y}[e^{j\tilde{\omega}_1}] \\ \mathbf{Y}[e^{j\tilde{\omega}_2}] \\ \vdots \\ \mathbf{Y}[e^{j\tilde{\omega}_K}] \end{bmatrix}}_{\triangleq \mathbf{Y}_K} = \underbrace{\begin{bmatrix} \mathbf{H}[e^{j\tilde{\omega}_1}] & & & \\ & \mathbf{H}[e^{j\tilde{\omega}_2}] & & \\ & & \ddots & \\ & & & \mathbf{H}[e^{j\tilde{\omega}_K}] \end{bmatrix}}_{\triangleq \mathbf{H}_K} \underbrace{\begin{bmatrix} \mathbf{U}[e^{j\tilde{\omega}_1}] \\ \mathbf{U}[e^{j\tilde{\omega}_2}] \\ \vdots \\ \mathbf{U}[e^{j\tilde{\omega}_K}] \end{bmatrix}}_{\triangleq \mathbf{U}_K}$$

non-zero at $\tilde{\omega}_1$ locations $U[e^{j\tilde{\omega}_1}] \in R^{m\times 1}$

non-zero at $\tilde{\omega}_K$ locations $U[e^{j\tilde{\omega}_K}] \in R^{m\times 1}$

Complex-valued LASSO

$$\widehat{\mathbf{U}}_K = \underset{\mathbf{U}_K \in \mathbb{C}^{mK\times 1}}{\arg\min} \left\{ \frac{1}{2} \|\mathbf{Y}_K - \mathbf{H}_K \mathbf{U}_K\|_2^2 + \lambda \|\mathbf{U}_K\|_1 \right\}$$

Empirically determining # of freq. (K)

vector valued FFT over a grid of frequencies

$$\widetilde{\mathbf{Y}}[q] \triangleq \frac{2}{N} \sum_{k=L}^{N-1+L} \mathbf{y}[k] e^{-j\frac{2\pi l k}{N}} \quad (q = 0, 1, \ldots, N-1)$$

$$\tilde{\omega}_l = 2\pi l/N$$

$$S = \{l : \|\widetilde{\mathbf{Y}}[l]\|_\infty > \tau\}, \text{ where } \tau > 0 \qquad K = |S|$$

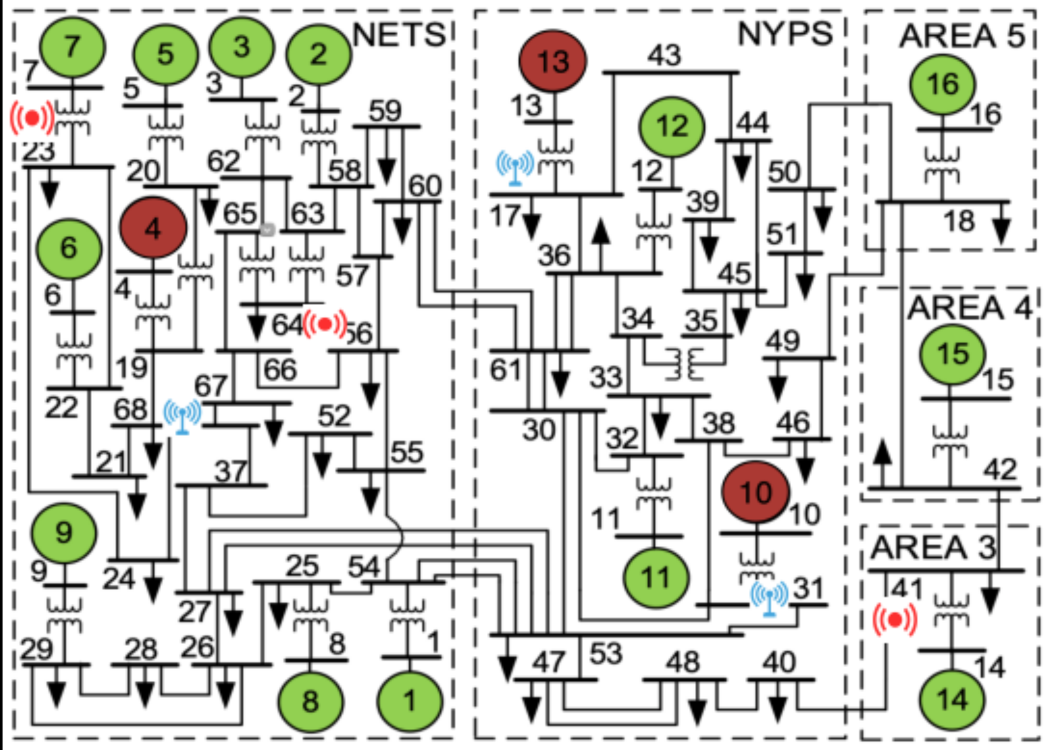
❖ measurements are first filtered to attenuate sensor noise and are windowed (Hamming) to reduce spectral leakage.

❖ τ is a user-defined threshold parameter

❖ we solved C-LASSO using complex-valued coordinate descent

SIMULATION RESULTS

IEEE NETS/NYPS 16 machine, 68 bus system



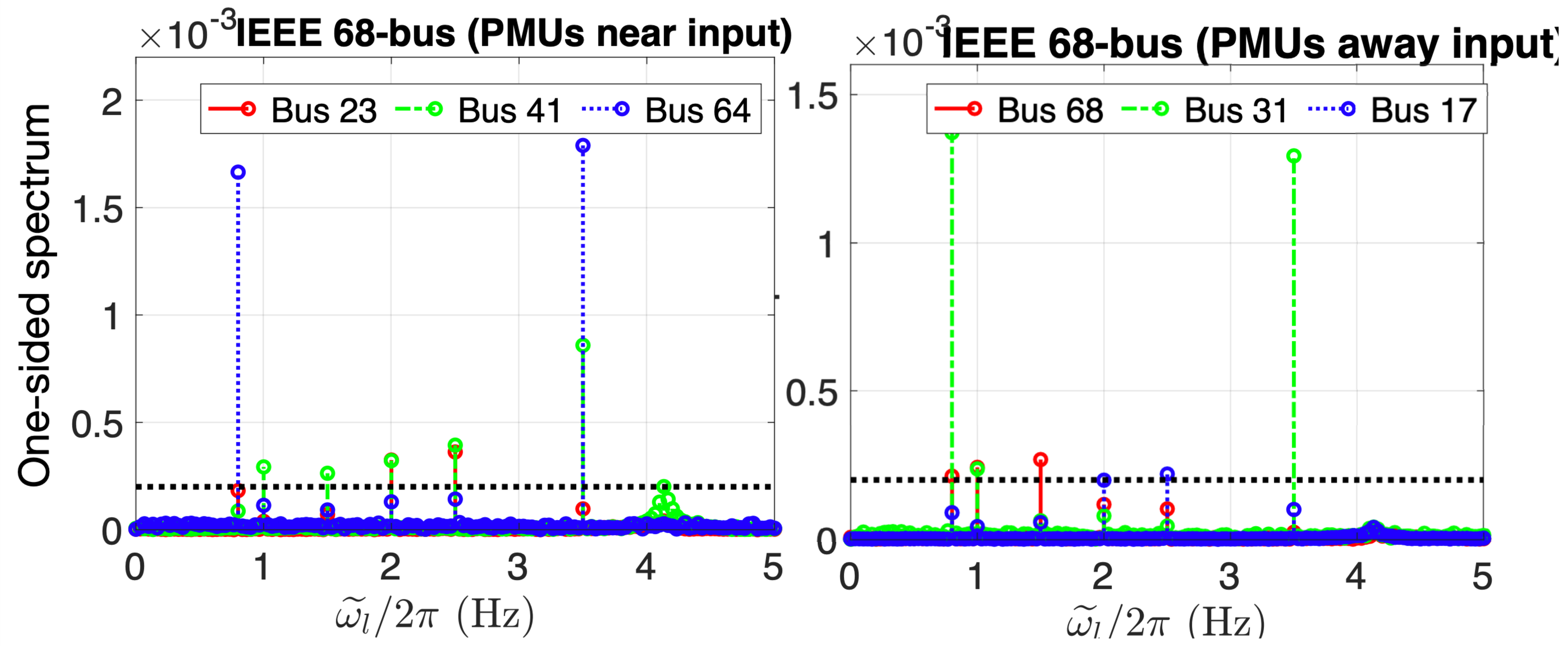
green bus: gen (10 dim state)

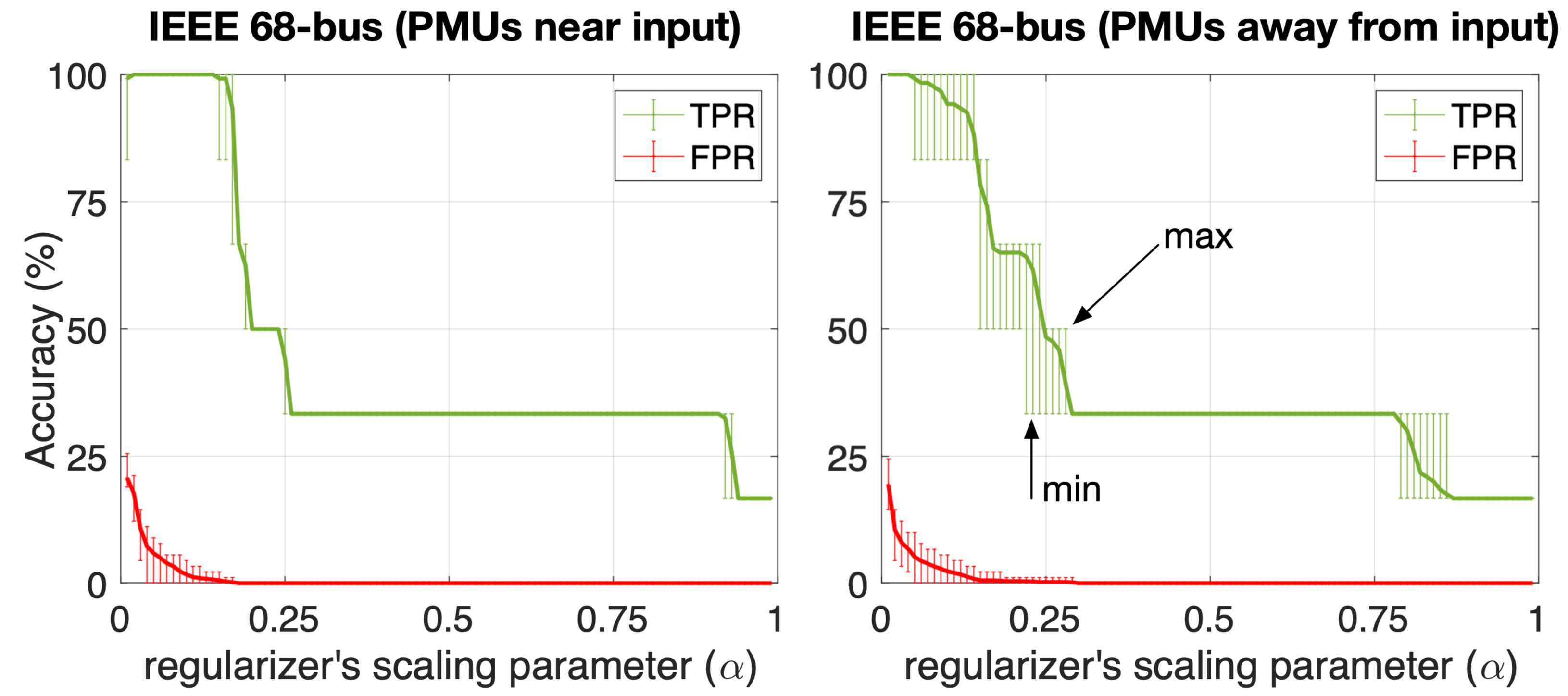
red bus {4, 13, 10} FO enters

PMU locations

blue: {31, 17, 67} → near

red: {64, 23, 41} → far





TPR (true +ve rate)= $\frac{correct\ non-zero\ \hat{U}}{true\ non-zero\ U}$

FPR (false +ve rate)= $\frac{incorrect\ non-zero\ \hat{U}}{true\ non-zero\ U}$

$\lambda = \alpha(\lambda_{max}),\ \alpha \in (0,1)$

BENEFITS AND POTENTIAL USES

❖ C-LASSO localizes sources and input parameters (amp, freq, phase) by exploiting sparsity in space and fourier-domain

❖ Rigorous guarantees can be provided on the estimation performance (scan QR code on the top-left)

❖ Future: load dynamics, DER-based harmonic models

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